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I. Solution by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

The system is not acted upon by any external force. Hence the center of gravity is stationary. Let I be the moment of inertia of the board about its center, m the mass of the board, m' that of the boy, r the distance of center of system from center of board, ω the angular velocity of board. Then $I\omega^2+mr^2\omega^2=m'v^2$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $AC=a$ be the radius of board, A the starting point of the boy, m his mass, P the mass of the board, G the center of gravity of the system at starting.

Since there are no horizontal forces *external* to the system, by D'Alembert's principle, the center of gravity of the system must move downwards; but this is impossible.

$\therefore$ The equation of motion of the center of gravity is, $m\dfrac{d^2\bar{x}}{dt^2}=0$.

$\therefore \dfrac{d\bar{x}}{dt}=0$, since both boy and board start from rest.

$\therefore \bar{x}=$a constant. $\therefore$ The center of gravity of the system remains unaltered throughout the motion.

Hence since the distance between the center of the board and boy remain constant, the distances of the center of gravity from the boy and the center of the board respectively will remain constant.

$\therefore C$ describes a circle with $GC=\dfrac{ma}{m+P}$ as radius.

$\therefore A$ describes a circle with $AG=\dfrac{Pa}{m+P}$ as radius.

$\therefore$ both center of board and boy describe circles. If $m=P$ these circles will be equal and have AC for diameter.

MISCELLANEOUS.

69. Proposed by WILLIAM SYMMONDS, A. M., Professor of Mathematics and Astronomy, Pacific College, Santa Rosa, P. O., Sebastopol, Cal.

Find the locus of a point equi-distant from the circumferences of two fixed circles.

I. Solution by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Denote the two given circumferences by K_1 and K_2; take their centers at $(-\frac{1}{2}a, 0)$ and $(\frac{1}{2}a, 0)$; denote their radii by r_1 and r_2, r_1 not being less than r_2. Let (x, y) denote a point in the desired locus.

Then if (x, y) is inside K_1 and inside K_2,

$$r_1-\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-\{r_2-\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}\}=0\ldots\ldots(1).$$

Or if (x, y) is outside K_1 and outside K_2,

$$\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-r_1-\{\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}-r_2\}=0,$$

which is the same as (1).

Similarly, if (x, y) is inside K_1 and outside K_2, or if (x, y) is outside K_1 and inside K_2,

$$r_1-\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-\{\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}-r_2\}=0 \ldots\ldots (2).$$

The desired locus is represented by the equation obtained by taking the product of (1) and (2). I am not aware of any general criterion as to the nature and limiting points of curves (or pieces of curve) whose equations are irrational. If either variable in the equation is an explicit function of the other, the nature of the curve, as well as the limiting points, can usually be detected by inspection; but considerable difficulty may be experienced in finding the nature and limiting points of the curve represented by an irrational equation in which x and y are implicit functions of each other.

Although aware that the method is exceedingly unsatisfactory, I will first include the curves or pieces of curve represented by the equations congeneric to (1) and (2) and afterward exclude such pieces thus incorrectly dragged in by the process of rationalization.

The equations, whose product make (1) rational, are:

$$r_1-\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-\{r_2-\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}\}=0 \ldots\ldots (1).$$

$$r_1-\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-\{r_2+\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}\}=A \ldots\ldots (3).$$

$$r_1+\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-\{r_2-\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}\}=B \ldots\ldots (4).$$

$$r_1+\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-\{r_2+\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}\}=C \ldots\ldots (5).$$

The equations, whose product make (2) rational, are

$$r_1-\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-\{\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}-r_2\}=0 \ldots\ldots (2).$$

$$r_1-\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}+\{\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}+r_2\}=D \ldots\ldots (6).$$

$$r_1+\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}-\{\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}-r_2\}=E \ldots\ldots (7).$$

$$r_1+\sqrt{[(x+\tfrac{1}{2}a)^2+y^2]}+\{\sqrt{[(x-\tfrac{1}{2}a)^2+y^2]}+r_2\}=F \ldots\ldots (8).$$

The product of (1), (3), (4), and (5) will represent all lines or parts of lines the points of which satisfy *any one of them with its right hand member put equal to zero*: for if the right hand member of any of them, say (3) was zero, and the right hand member of (1) was, say G, precisely *the same product would be* *obtained*; a similar statement may be made with reference to the product of (2), (6), (7), and (8), although in the case of (8) no points would be obtained by making the right hand member equal to zero, since the sum of two positive distances cannot equal zero.

Denote the difference of the radii (r_1-r_2) by d, and the sum (r_1+r_2) by s. The product of the first four equations is

$$4(a^2-d^2)x^2-4d^2y^2=d^2(a^2-d^2)\ldots\ldots(9).$$

And the product of the second four equations is

$$4(a^2-s^2)x^2-4s^2y^2=s^2(a^2-s^2)\ldots\ldots(10).$$

It will be remembered that a is the distance between the centers of the two given circles, d the difference, and s the sum of their radii. In eliminating from (9) the points not satisfying (1), and from (10) the points not satisfying (2), I will consider a to vary.

I. $a<d<s$ or K_2 is wholly inside K_1; in this case both (9) and (10) are equations of ellipses; (9) is the locus of centers of circles tangent internally to K_1 and enclosing K_2; these points satisfy (3) put equal to zero. Equation (10) is the locus of centers of circles tangent internally to K_1 and externally to K_2; all such points satisfy (2) and hence (10) is the equation of the desired locus.

II. If $a=d$ and $a<s$, K_2 is inside K_1 and internally tangent to it; in this case, equation (9) reduces to $y=0$ or the axis of X. From $x=+\infty$ to $x=+\frac{1}{2}a$, the points satisfy (1); from $x=+\frac{1}{2}a$ to $x=-\frac{1}{2}a$, the points satisfy (3) put equal to zero; from $x=-\frac{1}{2}a$ to $x=-\infty$, the points satisfy (5) put equal to zero. Equation (10) as in Case I. The locus consists then of (9), from $x=+\infty$ to $x=+\frac{1}{2}d$, and of (10).

III. If $a>d$ and $a<s$, K_1 and K_2 intersect; in this case equation (9) represents a hyperbola, all the points on the right hand branch satisfying (1), and the points on the left hand branch satisfying (5) put equal to zero. Equation (10) as in Cases I and II. The locus consists of (9), $x=+\infty$ to $x=+\frac{1}{2}d$, and of (10).

IV. If $a<d$ and $a=s$, K_1 and K_2 are tangent externally. Equation (9) as in Case III. Equation (10) reduces to $y=0$, or the axis of X. From $x=+\infty$ to $x=+\frac{1}{2}a$, the points satisfy (6); from $x=+\frac{1}{2}a$ to $-\frac{1}{2}a$, the points satisfy (2); and from $x=-\frac{1}{2}a$ to $-\infty$, the points satisfy (7). The locus consists of (9) from $x=+\infty$ to $x=+\frac{1}{2}d$, and of (10) from $x=+\frac{1}{2}a$ to $x=-\frac{1}{2}a$.

V. If $a>d$ and $a>s$, K_1 and K_2 are exterior to each other. Equation (9) as in Cases III and IV. Equation (10) also represents a hyperbola, the points on the right hand branch satisfying (6) put equal to zero, and the points on the left hand branch satisfying (7) put equal to zero. The locus consists of (9) from $x=+\infty$ to $x=+\frac{1}{2}d$.

In many cases the limiting points may be found by equating the radical to zero if it is of the second degree as in this case. But this is not true as a general proposition. For the right hand branch of the hyperbola (9) in III, IV, and V crosses the circle at $x=+\frac{1}{2}a$ and is inside from $x=+\frac{1}{2}a$ to $x=+\frac{1}{2}d$. The left hand branch begins at $x=-\frac{1}{2}d$ and leaves the circle at $x=-\frac{1}{2}a$. Between the limits suggested by equating the radicals to zero ($x=+\frac{1}{2}a$ and $x=-\frac{1}{2}a$) there are two pieces of curve, one of which contains points filling the given conditions, the other containing points violating them. That the tentative methods adopted by me are wholly unsatisfactory I freely admit, and yet I feel sure that graphical methods will be of great aid in establishing a theory of irrational equations.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

Let $AF=R$, $BE=r$, $CF=CE=t$.

Then when B is inside of circle A we have $BC+AC=r+t+R-t=R+r$.

$\therefore$ The locus of C, the point equidistant from both circumferences, is an ellipse, foci A, B.

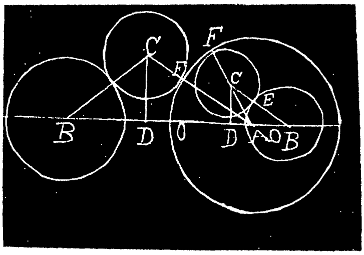
When B is without A we have $AC-BC=R+t-r-t=R-r$.

$\therefore$ The locus of C is a hyperbola, foci A, B. Also as follows:

Let the mid-point O, between A and B be origin, $AB=2a$, $OD=x$, $CD=y$.

$\therefore$ $BC^2=(r+t)^2=(a\pm x)^2+y^2$....(1).

$AC^2=(R\mp t)^2=(x\mp a)^2+y^2$...(2).



Eliminating t between (1) and (2) we get

$$r\pm R\mp\sqrt{[(x\mp a)^2+y^2]}=\sqrt{[(x\pm a)^2+y^2]},\text{ or }\frac{4x^2}{(R\pm r)^2}+\frac{4y^2}{(R\pm r)^2-4a^2}=1.$$

This equation shows that when one circle is within the other $(R+r)>2a$ and the locus is an ellipse with semi-axes $\frac{1}{2}(R+r)$ and $\frac{1}{2}\sqrt{[(R+r)^2-4a^2]}$.

When circles are external the locus is an hyperbola with semi-axes $\frac{1}{2}(R-r)$ and $\frac{1}{2}\sqrt{[4a^2-(R-r)^2]}$.

If the circles intersect, for internal contact to one circle and external to the other we have the same ellipse; for both internal and both external we have the same hyperbola as above.

Also solved by *GUY B. COLLIER*, *CHARLES C. CROSS*, *WALTER H. DRANE*, *ALOIS F. KOVARIK*, *W. W. LANDIS*, *J. SHEFFER*, and *COOPER D. SCHMITT*.

DIOPHANTINE ANALYSIS.

73. **Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Find integral values for x and y in $\begin{pmatrix}2x^2-y^2=\square\\2y^2-x^2=\square\end{pmatrix}$.